

Comparative Analysis of Finite Field-dependent BRST Transformations*

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Abstract

We present a review of our recent study [1, 2, 3, 4, 5, 6], in which the concept of finite field-dependent BRST and BRST-antiBRST transformations for gauge theories was introduced, and their properties investigated. An algorithm of exact calculation for the Jacobian of a respective change of variables in the path integral is presented. Applications to the Yang–Mills theory and Standard Model, in view of infra-red (Gribov) peculiarities, are discussed.

1 Introduction

BRST transformations [7, 8] for gauge theories in Lagrangian formalism were first examined in the capacity of *field-dependent* (FD) BRST transformations within the field-antifield approach [9] in order to prove the independence from small gauge variations (expressed through the gauge fermion ψ) of the path integral Z_ψ : $Z_\psi = Z_{\psi+\delta\psi}$, with the choice $\mu = -\frac{i}{\hbar}\delta\psi$ for the Grassmann-odd parameter of FD BRST transformations. Originally introduced as the case of a special $N = 1$ SUSY transformation, being a change of the field variables ϕ^A ,

$$\phi^A \rightarrow \phi^{A'} = \phi^A + \delta_\mu \phi^A, \quad \mathcal{I}_\phi^\psi = d\phi \exp \left\{ \frac{i}{\hbar} S_\psi(\phi) \right\}, \quad Z_\psi = \int \mathcal{I}_\phi^\psi, \quad (1)$$

in the integrand \mathcal{I}_ϕ^ψ with a quantum action $S_\psi(\phi)$, BRST transformations were extended, by means of antiBRST transformations [10, 11] in Yang–Mills theories, to $N = 2$ BRST-antiBRST transformations (in Yang–Mills [12] and general gauge theories [13]), which were associated with an $\text{Sp}(2)$ -doublet of Grassmann-odd parameters, μ_a , $a = 1, 2$.

The concept of *finite* FD BRST transformations was introduced by Joglekar and Mandal [14] in Yang–Mills theories, as a sequence of infinitesimal FD BRST transformations (with a numeric parameter κ), in order to prove the gauge-independence of the path integral within the family of R_ξ -gauges and their non-linear deformations in the field variables.

The study [15] by a group of Brazilian researchers (see also the references therein) suggested an analysis of so-called *soft BRST symmetry breaking* in Yang–Mills theories, with reference to the Gribov problem [16] in the long-wave spectra of field configurations, which

*Talk presented at SQS'15, 03 August – 08 August, 2015, at JINR, Dubna, Russia

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also involves the Zwanziger proposal [17] for a horizon functional joined additively to a BRST invariant quantum action. The study of [15], with the related scope of problems, caught the attention of one the authors of the present article (A.A.R.), who subsequently (March, 2011) turned it to the attention of P.M. Lavrov and then O. Lechtenfeld. The resulting study [18] of this problem within the field-antifield formalism suggested an equation for the BRST non-invariant addition $M(\phi, \phi^*)$ to the quantum action $S_\psi(\phi, \phi^*)$ of a general gauge theory. The validity of this equation preserves the gauge-independence of the vacuum functional $Z_{\psi,M}(0)$ [see (5) for a definition] and effective action, depending on external antifields, $\Gamma_M = \Gamma_M(\phi, \phi^*)$, and evaluated on the extremals $\Gamma_M \overleftarrow{\delta}_{\phi^A} = \Gamma_M \overleftarrow{\partial}_A = \Gamma_{M,A} = 0$,

$$\begin{aligned} & \left[M_A(\frac{\hbar}{i} \overrightarrow{\partial}_{(J)}, \phi^*) \left(\overrightarrow{\partial}^{*A} - \frac{i}{\hbar} M^{*A}(\frac{\hbar}{i} \overrightarrow{\partial}_J, \phi^*) \right) \delta\psi(\frac{\hbar}{i} \overrightarrow{\partial}_{(J)}) + \delta M(\frac{\hbar}{i} \overrightarrow{\partial}_{(J)}, \phi^*) \right] Z_{\psi,M}(J, \phi^*) = 0 \quad (2) \\ \implies & Z_{\psi,M}(0) = Z_{\psi+\delta\psi, M+\delta M}(0) \text{ and } \delta\Gamma_M|_{\Gamma_{M,A}=0} = 0, \text{ where } \Gamma_M = \frac{\hbar}{i} \ln Z - J_A \phi^A, \quad (3) \end{aligned}$$

where it is assumed that $[M_A, M^{*A}] \equiv [M \overleftarrow{\partial}_A, \overrightarrow{\partial}^{*A} M]$, and $\phi^A = \frac{\hbar}{i} \overrightarrow{\partial}_{(J)}^A \ln Z$ are average fields, having the same form when the horizon functional $H(A)$ for YM fields $A^{\mu n}(x)$ is used as $M(\phi, \phi^*)$. In terms of the vacuum expectation value, in the presence of external sources J_A , and with a given gauge ψ , relation (2) acquires the form,¹ $\frac{\overrightarrow{\partial} S_\psi}{\delta \phi_A^*} \equiv \overrightarrow{\partial}^{*A} S_\psi$,

$$\langle \delta M + M \overleftarrow{s} \frac{i}{\hbar} \delta\psi(\phi) \rangle = \langle \delta M - M \overleftarrow{s} \mu(\delta\psi) \rangle = 0, \text{ where } \overleftarrow{s} = \overleftarrow{\partial}_A S_\psi \overrightarrow{\partial}^{*A} S_\psi : \delta_\mu \phi^A \equiv \phi^A \overleftarrow{s} \mu, \quad (4)$$

where \overleftarrow{s} is the generator of BRST transformations (Slavnov generator in YM theories). This fact was established in [1]. In December 2011, one of the authors (A.A.R.) drew the attention of his coauthors (P.M. Lavrov) in [18] to the research [19] which attempted to use FD BRST transformations [14] for relating the vacuum functionals in YM and Gribov–Zwanziger (GZ) under different gauges. An explicit calculation of the functional Jacobian for a change of variables induced by FD BRST transformations in YM theories with a finite parameter μ was made in [20], to establish the gauge-independence of Z_ψ under a finite change of the gauge, $\psi \rightarrow \psi + \Delta\psi$, and afterwards in [21], to solve equation (4) with $M(\phi) = H(A)$, in a way different from anticanonical transformations, as compared to [18].

The present article reviews the study of finite BRST and BRST-antiBRST transformations (including the case of field-dependent parameters) and the way they influence the properties of the quantum action and path integral in conventional quantization. The article has the following organization. Section 2 presents the definitions of finite BRST and BRST-antiBRST transformations. The algorithm for calculating the functional determinants related to these transformations is briefly examined in Section 3. The implications of this calculation to the quantum structure of gauge theories are presented in Section 4.

We use the DeWitt condensed notation and the conventions of [1, 2], e.g., we use $\epsilon(F)$ for the value of Grassmann parity of a quantity F . Derivatives with respect to (anti)field variables ϕ^A, ϕ_A^* and sources J_A are denoted by $\overleftarrow{\partial}^A, (\overrightarrow{\partial}_A^*)$ and $\overrightarrow{\partial}_{(J)}^A$. The raising and lowering of Sp(2) indices, $(\overleftarrow{s}^a, \overleftarrow{s}_a) = (\varepsilon^{ab} \overleftarrow{s}_b, \varepsilon_{ab} \overleftarrow{s}^b)$, are carried out by a constant antisymmetric tensor $\varepsilon^{ab}, \varepsilon^{ac} \varepsilon_{cb} = \delta_b^a, \varepsilon^{12} = 1$.

¹In fact, the horizon functional in the family of R_ξ -gauges for small ξ was found explicitly in [18], see Eq. (5.20) therein, by using FD BRST transformations with a small odd-valued parameter.

2 Proposals for Finite BRST Transformations

The problem of softly broken BRST symmetry (SB BRST) in general gauge theories was solved in [1] on a basis of finite FD (“gauged” in the terminology of [1]) BRST transformations (invariance transformations for the integrand in (5) at $J = M = 0$) with finite odd-valued parameters $\mu(\phi, \phi^*)$ depending on external antifields ϕ_A^* , $\epsilon(\phi_A^*) + 1 = \epsilon(\phi^A) = \epsilon_A$, and fields ϕ^A whose contents include the classical fields A^i , $i = 1, \dots, n$, with gauge transformations $\delta A^i = R_\alpha^i(A)\xi^\alpha$, $\alpha = 1, \dots, m < n$, the ghost, antighost, and Nakanishi–Lautrup fields $C^\alpha, \bar{C}^\alpha, B^\alpha$, $\epsilon(A^i, \xi^\alpha, C^\alpha, \bar{C}^\alpha, B^\alpha) = (\epsilon_i, \epsilon_\alpha, \epsilon_\alpha + 1, \epsilon_\alpha + 1, \epsilon_\alpha)$, as well as the additional towers of fields depending on the (ir)reducibility of the theory. The generating functional of Green’s functions depending on external sources J_A , $\epsilon(J_A) = \epsilon_A$, with an SB BRST symmetry term M , $\epsilon(M) = 0$, is given by

$$Z_{\psi, M}(J, \phi^*) = \int d\phi \exp \left\{ \frac{i}{\hbar} S_\psi(\phi, \phi^*) + M(\phi, \phi^*) + J_A \phi^A \right\}, \text{ with } \overleftarrow{s}_e = \overleftarrow{\partial}_A \overrightarrow{\partial}^{*A} S_\psi \equiv \overleftarrow{\partial}_A S_\psi^{*A}, \quad (5)$$

where the generator \overleftarrow{s}_e , $\phi^A \overleftarrow{s}_e = S_\psi^{*A}(\phi, \phi^*)$, reduces at $\phi^* = 0$ to the usual generator \overleftarrow{s} of (FD) BRST transformations, $\delta_\mu \phi^A = S_\psi^{*A}(\phi, 0)\mu$, and fails to be nilpotent, due to the quantum master equation for S_ψ ,

$$\left[\Delta \exp \left\{ \frac{i}{\hbar} S_\psi \right\} = 0 \iff S_\psi \overleftarrow{\partial}_A \overrightarrow{\partial}^{*A} S_\psi = i\hbar \Delta S_\psi \right] \Rightarrow (\overleftarrow{s}_e)^2 = \overleftarrow{\partial}_A (S_\psi^{*A} \overleftarrow{\partial}_B) S_\psi^{*B} \neq 0, \quad (6)$$

with $\Delta = (-1)^{\epsilon_A} \overrightarrow{\partial}_A \overrightarrow{\partial}^{*A}$. The Jacobian induced by a change of variables² $\phi^A \rightarrow \phi'^A = \phi^A(1 + \overleftarrow{s}_e \mu)$ was calculated originally in [1]:

$$\text{Sdet} \left\| \phi'^A \overleftarrow{\partial}_B \right\| = \exp \left\{ \text{Str} \ln \left(\delta_B^A + (S_\psi^{*A} \mu) \overleftarrow{\partial}_B \right) \right\} = \exp \left\{ \text{Str} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left((S_\psi^{*A} \mu) \overleftarrow{\partial}_B \right)^n \right\} \quad (7)$$

$$\implies \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \text{Str} \left((S_\psi^{*A} \mu) \overleftarrow{\partial}_B \right)^n = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (\mu \overleftarrow{s}_e)^n - \sum_{n=2}^{\infty} (-1)^n (\mu \overleftarrow{s}_e)^{n-2} \mu_A (S_\psi^{*A} \overleftarrow{s}_e) \mu \quad (8)$$

$$+ (\Delta S_\psi) \mu \implies \text{Sdet} \left\| \phi'^A \overleftarrow{\partial}_B \right\| = (1 + \mu \overleftarrow{s}_e)^{-1} \left\{ 1 + \overleftarrow{s}_e \mu \right\} \left\{ 1 + (\Delta S_\psi) \mu \right\}, \quad (9)$$

under a suitable condition of convergence for the series in (8), and reduces, in a rank-1 theory with a closed gauge algebra, to the form

$$[\Delta S_\psi, \overleftarrow{s}^2] = [0, 0], \text{ where } \overleftarrow{s}_e = \overleftarrow{s} \implies \text{Sdet} \left\| \Phi'^A \overleftarrow{\partial}_B \right\| = (1 + \mu \overleftarrow{s})^{-1}, \quad (10)$$

which is the same as in YM theories.

The construction of finite BRST-antiBRST Lagrangian transformations solving the same problem within a suitable quantization scheme (starting from YM theories), is problematic in view of the BRST-antiBRST non-invariance of the gauge-fixed quantum action S_F in a form more than linear in μ_a , $S_F(g_l(\mu_a)\phi) = S_F(\phi) + O(\mu_1 \mu_2)$, with the gauge condition encoded by a gauge boson $F(\phi)$. This problem was solved in January 2014, by constructing finite BRST-antiBRST transformations in a group form, $\{g(\mu_a)\}$, using an appropriate set of variables Γ^p , according to [2]

$$\{G(g(\mu_a)\Gamma) = G(\Gamma) \text{ and } G \overleftarrow{s}^a = 0\} \Rightarrow g(\mu_a) = 1 + \overleftarrow{s}^a \mu_a + \frac{1}{4} \overleftarrow{s}^a \overleftarrow{s}_a \mu^2 = \exp \{ \overleftarrow{s}^a \mu_a \}, \quad (11)$$

²In the case $\mu \overleftarrow{s}_e \neq 0$, the set $\{g(\mu)\}$, for $\phi' = \phi g(\mu)$, cannot be presented as Lie group elements: $g(\mu) \neq \exp(\overleftarrow{s}_e \mu)$.

for an arbitrary regular functional $G(\Gamma)$, where $\mu^2 \equiv \mu_a \mu^a$, and $\overleftarrow{s}^a, \overleftarrow{s}^2 \equiv \overleftarrow{s}^a \overleftarrow{s}_a$ are the generators of BRST-antiBRST and mixed BRST-antiBRST transformations in the space of Γ^p . These transformations, however, cannot be presented as group elements (in terms of an exp-like relation) for an $\text{Sp}(2)$ doublet μ_a which is not closed under FD BRST-antiBRST transformations: $\mu_a \overleftarrow{s}_b \neq 0$.

In YM theories, the quantum action $S_F(\phi)$ in R_ξ -like gauges [given by a bosonic gauge functional $F(\phi)$] and the finite BRST-antiBRST transformations are constructed using an explicit form of BRST-antiBRST generators in the space of fields $\phi^A = (A^i, C^\alpha, \bar{C}^\alpha, B^\alpha)$, being identical with those of the Faddeev–Popov quantization rules [22] and organized in $\text{Sp}(2)$ -symmetric tensors, $(A^i, C^{\alpha a}, B^\alpha) = (A^{\mu m}, C^{ma}, B^m)$, as follows [2]:

$$S_F(\phi) = S_0(A) - \frac{1}{2} F_\xi \overleftarrow{s}^2, \quad S_0(A) = -\frac{1}{4} \int d^D x G_{\mu\nu}^m G^{m\mu\nu}, \quad G_{\mu\nu}^m = \partial_{[\mu} A_{\nu]}^m + f^{mnl} A_\mu^n A_\nu^l, \quad (12)$$

$$F_\xi(\phi) = \frac{1}{2} \int d^D x \left(-A_\mu^m A^{m\mu} + \frac{\xi}{2} \varepsilon_{ab} C^{ma} C^{mb} \right), \quad (13)$$

$$\Delta A_\mu^m = D_\mu^{mn} C^{na} \mu_a - \frac{1}{2} \left(D_\mu^{mn} B^n + \frac{1}{2} f^{mnl} C^{la} D_\mu^{nk} C^{kb} \varepsilon_{ba} \right) \mu^2, \quad (14)$$

$$\Delta B^m = -\frac{1}{2} \left(f^{mnl} B^l C^{na} + \frac{1}{6} f^{mnl} f^{lrs} C^{sb} C^{ra} C^{nc} \varepsilon_{cb} \right) \mu_a, \quad (15)$$

$$\Delta C^{ma} = \left(\varepsilon^{ab} B^m - \frac{1}{2} f^{mnl} C^{la} C^{nb} \right) \mu_b - \frac{1}{2} \left(f^{mnl} B^l C^{na} + \frac{1}{6} f^{mnl} f^{lrs} C^{sb} C^{ra} C^{nc} \varepsilon_{cb} \right) \mu^2. \quad (16)$$

These relations are inherited from the non-Abelian gauge transformations in terms of a covariant derivative,

$$\delta A_\mu^m(x) = D_\mu^{mn}(x) \zeta^n(x) = \int d^D y R_\mu^{mn}(x; y) \zeta^n(y), \quad \text{where } i = (\mu, m, x), \alpha = (n, y), \quad (17)$$

where the generator of gauge transformations $R_\mu^{mn}(x; y)$ leaves the classical action S_0 invariant with accuracy up to the first order in arbitrary functions ζ^n : $S_0(A + \delta A) = S_0(A) + o(\zeta^n)$; the metric tensor is $\eta_{\mu\nu} = \text{diag}(-, +, \dots, +)$, and f^{lmn} are the totally antisymmetric $su(\hat{N})$ structure constants, $l, m, n = 1, \dots, \hat{N}^2 - 1$. The $N = 2$ BRST-antiBRST invariant action for YM theories coincides with the $N = 1$ BRST invariant action only in Landau gauge, $\xi = 0$, whereas in the gauges $F_{\xi \neq 0}$ including the Feynmann gauge, $\xi = 1$, the action S_F contains terms quartic in the ghosts C^{ma} , leading thereby to additional vertices in the Feynmann integrals. This is a property of quantization with a global SUSY symmetry higher than $N = 1$ BRST symmetry.

In general gauge theories, such as reducible ones (for higher-spin field theories, see [23, 24] and references therein), or those with an open gauge algebra (as in some SUSY models), the corresponding space of triplectic variables $\Gamma_{tr}^p = (\phi^A, \phi_{Aa}^*, \bar{\phi}_A, \pi^{Aa}, \lambda^A)$ in the $\text{Sp}(2)$ -covariant Lagrangian quantization method [13] contains, in addition to ϕ^A , three sets of antifields $\phi_{Aa}^*, \bar{\phi}_A, \epsilon(\phi_{Aa}^*, \bar{\phi}_A) = (\epsilon_A + 1, \epsilon_A)$, as sources to BRST, antiBRST and mixed BRST-antiBRST transformations, and three sets of Lagrangian multipliers $\pi^{Aa}, \lambda^A, \epsilon(\pi^{Aa}, \lambda^A) = (\epsilon_A + 1, \epsilon_A)$, introducing the gauge. The corresponding generating functional of Green's functions, $Z_F(J)$,

$$Z_F(J) = \int d\Gamma \exp \left\{ (i/\hbar) \left[W + \phi_a^* \pi^a + \bar{\phi} \lambda - \frac{1}{2} F \overleftarrow{U}^2 + J\phi \right] \right\}, \quad \overleftarrow{U}_a = \overleftarrow{\partial}_A \pi^{Aa} + \varepsilon^{ab} \overleftarrow{\partial}_{Ab}^{(\pi)} \lambda^A \quad (18)$$

is invariant, at $J = 0$, with respect to finite BRST-antiBRST transformations (for constant

μ_a) in the space of Γ_{tr}^p , which are given by (11), however, with the functional $G_{tr} = G(\Gamma_{tr}^p)$:

$$\Gamma_{tr}^p \rightarrow \Gamma_{tr}'^p = \Gamma_{tr}^p (1 + \overleftarrow{s}^a \mu_a + \frac{1}{4} \overleftarrow{s}^2 \mu^2) \equiv \Gamma_{tr}^p g(\mu_a) \implies \mathcal{I}_{\Gamma_{tr}'^p g(\mu_a)}^{(F)} = \mathcal{I}_{\Gamma_{tr}^p}^{(F)} \text{ for } Z_F = \int \mathcal{I}_{\Gamma_{tr}^p}^{(F)}, \quad (19)$$

where $\overleftarrow{s}^a = \left(\overleftarrow{\partial}_A, \overleftarrow{\partial}_{(\phi^*)}^{Aa}, \overleftarrow{\partial}_{(\bar{\phi})}^A, \overleftarrow{\partial}_{Ab}^{(\pi)} \right) \left(\pi^{Aa}, W_{,A}(-1)^{\epsilon_A}, \varepsilon^{ab} \phi_{Ab}^* (-1)^{\epsilon_A+1}, \varepsilon^{ab} \lambda^A \right)^T, \{\overleftarrow{s}^a, \overleftarrow{s}^b\} \neq 0$,

$$\text{provided that } \left(\Delta^a + (\imath/\hbar) \varepsilon^{ab} \phi_{Ab}^* \overrightarrow{\partial}_{(\bar{\phi})}^A \right) \exp \left\{ \frac{\imath}{\hbar} W \right\} = 0, \text{ for } \Delta^a = (-1)^{\epsilon_A} \overrightarrow{\partial}_A \overrightarrow{\partial}^{*Aa}, \quad (20)$$

with respective classical action $S_0(A)$ being the boundary condition for the equation (20) on the bosonic functional W : $W(\phi, \phi_a^*, \bar{\phi})|_{\phi_a^* = \bar{\phi} = 0} = S_0(A)$. The restricted generators $\overleftarrow{U}^a = \overleftarrow{s}^a|_{\phi, \pi, \lambda}$ are nilpotent and satisfy the algebra $\{\overleftarrow{U}^a, \overleftarrow{U}^b\} = 0$.

3 Jacobians of Finite $N = 1, 2$ BRST Transformations

The Jacobian (9) allows one to solve the problem of SB BRST symmetry in general gauge theories [1] and was examined in detail [5] for an equivalent representation of $Z_{\psi, M}(J, \phi^*)$ in (21), as well as for BRST transformations in an extended space which contains, besides ϕ^A , also internal (included in the path integral) antifields $\tilde{\phi}_A^*$ and Lagrangian multipliers λ^A to Abelian hypergauge conditions, $\mathcal{G}_A(\phi, \phi^*) = \phi_A^* - \psi(\phi) \overleftarrow{\partial}_A$:

$$Z_{\psi, M}(J, \phi^*) = \int d\Gamma \exp \left\{ \frac{\imath}{\hbar} S(\phi, \tilde{\phi}^*) + \mathcal{G}_A(\phi, \tilde{\phi}^{*A} + \phi^*) \lambda^A + M(\phi, \phi^*) + J\phi \right\}, \quad (21)$$

$$\text{with } \Gamma^p \rightarrow \Gamma^{p'} = \Gamma^p (1 + \overleftarrow{s} \mu), \text{ where } \Gamma^p \overleftarrow{s} = (\phi^A, \tilde{\phi}_A^*, \lambda^A) \overleftarrow{s} = (\lambda^A, S \overleftarrow{\partial}_A, 0) \quad (22)$$

$$\text{and } \text{Sdet} \left\| \Gamma^{p'} \overleftarrow{\partial}_q \Gamma \right\| = (1 + \mu \overleftarrow{s})^{-1} \left\{ 1 + (\Delta S) \mu \right\} + O(\mu \overleftarrow{s} \mu), \quad (23)$$

with a field-dependent $\mu(\Gamma)$ in (23), which reduces (for ϕ^* -independent $\mu(\Gamma)$: $\mu(\Gamma) \overleftarrow{\partial}^{*A} = 0$) to the Jacobian of [25], i.e., one without $O(\mu \overleftarrow{s} \mu)$. The bosonic functional $S(\phi, \phi^*)$ in (21) is a proper solution of (6), with the classical action S_0 as the boundary condition at $\phi^* = 0$.

For BRST-antiBRST transformations in YM theories, the technique of calculating the Jacobian was first examined for functionally-dependent parameters $\mu_a = \Lambda(\phi) \overleftarrow{s}_a$ with an even-valued functional Λ and was developed in [2], resulting in, $\phi'^A \equiv \phi^A g(\Lambda(\phi) \overleftarrow{s}_a)$,

$$J_{\Lambda(\phi) \overleftarrow{s}_a} = \text{Sdet} \left\| \phi'^A \overleftarrow{\partial}_B \right\| = \exp \{ \text{Str} \ln (\delta_B^A + M_B^A) \}, \text{ for } M_B^A = P_B^A + Q_B^A + R_B^A \quad (24)$$

$$= \phi^A \overleftarrow{s}^a (\mu_a \overleftarrow{\partial}_B) + \mu_a [(\phi^A \overleftarrow{s}^a) \overleftarrow{\partial}_B - \frac{1}{2} (\phi^A \overleftarrow{s}^2) (\mu^a \overleftarrow{\partial}_B)] (-1)^{\epsilon_A+1} + \frac{1}{4} \mu^2 (\phi^A \overleftarrow{s}^2 \overleftarrow{\partial}_B),$$

$$\text{Str}(P + Q + R)^n = \text{Str}(P + Q)^n + C_n^1 \text{Str} P^{n-1} R, \text{ for } C_n^k = n! / k! (n-k)!, \quad (25)$$

$$\text{Str}(P + Q)^n = \begin{cases} \text{Str} P^n + n \text{Str} P^{n-1} Q + C_n^2 \text{Str} P^{n-2} Q^2, & n = 2, 3, \\ \text{Str} P^n + n \sum_{k=0}^2 \text{Str} P^{n-k} Q^k + K_n \text{Str} P^{n-3} Q P Q, & n > 3 \end{cases} \quad (26)$$

$$\implies J_{\Lambda(\phi) \overleftarrow{s}_a} = \exp \left\{ \sum_{n=1} (-1)^{n-1} n^{-1} \text{Str} (P_B^A)^n \right\} = (1 - \frac{1}{2} \Lambda \overleftarrow{s}^2)^{-2}, \quad (27)$$

where $K_n = \left[\frac{n+1}{2} - 2 \right] C_n^1 + ((n+1) \bmod 2) C_{\lfloor \frac{n}{2} \rfloor}^1$, with $[x]$ being the integer part of $x \in \mathbb{R}$. The Jacobian (27) cannot be derived from the Jacobian (10) corresponding to FD BRST

transformations in YM theories.³ For functionally-independent FD parameters $\mu_a(\phi) \neq \Lambda \overleftarrow{s}_a$, the above algorithm (24)–(27) of calculating J_{μ_a} involves a generalization of (26) to the case $P^n \neq f^{n-1}P$, examined separately for odd and even $n > 3$, which leads to [6]

$$J_{\mu_a} = \exp \left\{ \text{tr} \sum_{n=1} (-1)^{n-1} n^{-1} \text{Str}(P_B^A)^n \right\} = \exp \{ -\text{tr} \ln(e + m) \}, \quad m_b^a = \mu_b \overleftarrow{s}^a, \quad (28)$$

where $(e)_b^a$ and tr denote δ_b^a and trace over $\text{Sp}(2)$ indices. The Jacobian (28) is generally not BRST-antiBRST exact; however, it reduces at $\mu_a = \Lambda \overleftarrow{s}_a$ to the Jacobian (27), due to

$$\text{tr} m_b^a = \text{tr} \Lambda \overleftarrow{s}_b \overleftarrow{s}^a = -(1/2) \text{tr} \delta_b^a \Lambda \overleftarrow{s}^2 \Rightarrow \text{tr} m^n = 2[-(1/2) \Lambda \overleftarrow{s}^2]^n \Rightarrow J_{\mu_a} = J_{\Lambda \overleftarrow{s}_a}. \quad (29)$$

In general gauge theories (18)–(20), the calculation of Jacobians induced by FD BRST-antiBRST transformations was first carried out in [3, 5] with functionally-dependent parameters $\mu_a = \Lambda(\phi, \pi, \lambda) \overleftarrow{U}_a$ and then in [6] with arbitrary parameters $\mu_a(\Gamma_{tr})$, including functionally-independent $\mu_a(\phi, \pi, \lambda)$. The result is given by

$$J_{\Lambda \overleftarrow{U}_a} = \text{Sdet} \left\| \left[\Gamma_{tr}^p g(\Lambda \overleftarrow{U}_a) \right] \overleftarrow{\partial}_q^\Gamma \right\| = \exp \left[-(\Delta^a W) \mu_a - \frac{1}{4} (\Delta^a W) \overleftarrow{s}_a \mu^2 \right] \left(1 - \frac{1}{2} \Lambda \overleftarrow{s}^2 \right)^{-2}, \quad (30)$$

$$J_{\mu_a(\phi, \pi, \lambda)} = \exp \left\{ -(\Delta^a W) \mu_a - \frac{1}{4} (\Delta^a W) \overleftarrow{s}_a \mu^2 - \text{tr} \ln(e + m) \right\}, \quad (31)$$

$$J_{\mu_a(\Gamma_{tr})} = J|_{\mu_a(\phi, \pi, \lambda) \rightarrow \mu_a(\Gamma_{tr})} \exp \left\{ \frac{1}{4} (\mu_a \overleftarrow{\partial}_p^\Gamma) [(e + m)^{-1}]_b^a (\Gamma_{tr}^p \overleftarrow{s}^2 \overleftarrow{s}^b) \mu^2 \right\}. \quad (32)$$

The second multiplier in (32) draws a difference between the Jacobians $J_{\mu_a(\phi, \pi, \lambda)}$ and $J_{\mu_a(\Gamma_{tr})}$, because \overleftarrow{s}_a are not reduced to the nilpotent \overleftarrow{U}_a as they act on Γ_{tr}^p . This result generalizes the Jacobian for $\mu_a(\phi, \pi, \lambda)$ obtained by a special Green function, using a t -parametric rescaling of the Lie equations in [25], and cannot be achieved by the prescription therein, due to the non-integrability condition $\overleftarrow{s}_a \overleftarrow{s}_b \overleftarrow{s}_c \neq 0$. For constant parameters μ_a , the Jacobians (30)–(32) are reduced to the same expression, being an \hbar -deformation of the classical master equations (20) and their differential consequences obtained by applying \overleftarrow{s}_a .

In generalized Hamiltonian formalism, the Jacobians of corresponding FD BRST-antiBRST transformations were calculated from first principles by the rules (24)–(28) in [4, 6], whereas the so-called *linearized* finite BRST-antiBRST transformations $\Gamma \rightarrow \Gamma' = \Gamma(1 + \overleftarrow{s}^a \mu_a)$ in YM and arbitrary first-class constraint dynamical systems, along with a calculation of Jacobians and a discussion of their implications on the quantum theory, were presented in [6].

4 Implications of Finite BRST Transformations

The proposals for $N = 1$ and $N = 2$ finite BRST transformations allow one to establish (in the case of constant μ and μ_a) the finite BRST (identical with the case of small μ) and BRST-antiBRST invariance of the integrand in (21) with a vanishing $N = 1, 2$ SB BRST symmetry term M , as well as in (18) for YM and general gauge theories.

³The corresponding Jacobian is not equal to the expression $J = \text{Sdet} \left\| \phi^{A'} \overleftarrow{\partial}_B \right\| = (1 + [\bar{\mu} \overleftarrow{s} \mu] \overleftarrow{s})^{-1} = 1 - \bar{\mu} \overleftarrow{s} \mu \overleftarrow{s} + o(\mu \bar{\mu})$, which is presented in [http://theor.jinr.ru/sqs15/Lavrov.pdf]. The just mentioned expression does not allow one to control the gauge independence of Z_F in (18) for infinitesimal FD parameters $(\mu_1, \mu_2) = (\mu, \bar{\mu})$, $(\overleftarrow{s}^1, \overleftarrow{s}^2) = (\hat{s}, \hat{\bar{s}})$, either in YM or in general gauge theories.

For FD parameters, finite BRST transformations allow one to obtain a new form of the Ward identity and to establish the gauge independence of the path integral under a finite change of the gauge, $\psi \rightarrow \psi + \psi'$, provided that the SB BRST symmetry term $M = M_\psi$ transforms to $M_{\psi+\psi'} = M_\psi(1 + \overleftarrow{s} \mu(\psi'))$, with $\mu(\psi')$ being a solution of a so-called compensation equation, as one makes a change of variables corresponding to an FD BRST transformation in the integrand of $Z_{\psi, M_\psi}(0, \phi^*)$:

$$Z_{\psi, M_\psi}(0, \phi^*) = Z_{\psi+\psi', M_{\psi+\psi'}}(0, \phi^*) \Rightarrow \psi'(\phi, \lambda|\mu) = \frac{\hbar}{i} \left[\sum_{n=1} \frac{(-1)^{n-1}}{n} (\mu \overleftarrow{s})^{n-1} \right] \mu. \quad (33)$$

The Ward identity, depending on the FD parameter $\mu(\psi') = -\frac{i}{\hbar} g(y) \psi'$, for $g(y) = 1 - \exp\{y\}/y$, $y \equiv (i/\hbar) \psi' \overleftarrow{s}$, and the gauge-dependence problem are described by the respective expressions [5]

$$\left\langle \left\{ 1 + \frac{i}{\hbar} [J_A \phi^A + M_\psi] \overleftarrow{s} \mu(\psi') \right\} (1 + \mu(\psi') \overleftarrow{s})^{-1} \right\rangle_{\psi, M, J} = 1 \text{ and } \langle (J_A \phi^A + M_\psi) \overleftarrow{s} \rangle_{\psi, M, J} = 0, \quad (34)$$

as one makes averaging with respect to $Z_{\psi, M_\psi}(J, \phi^*)$. The above equations are equivalent to those of [1, 18]

In turn, FD BRST-antiBRST transformations solve the same problem (allowing for the presence of an SB BRST-antiBRST symmetry term $M = M_F$ and not touching upon the unitarity issue [5, 27]) under a finite change of the gauge, $F \rightarrow F + F'$, provided that the term M_F transforms to $M_{F+F'} = M_F(1 + \overleftarrow{s}^a \mu_a(F') + \frac{1}{4} \overleftarrow{s}^2 \mu^2(F'))$, with $\mu_a(F'; \phi, \pi, \lambda) = \Lambda \overleftarrow{U}_a$ being a solution to the corresponding compensation equation, as one makes a change of variables corresponding to an FD BRST transformation in the integrand of $Z_F(0)$ from (18):

$$Z_F(0) = Z_{F+F'}(0) \Rightarrow F'(\phi, \pi, \lambda|\mu_a) = 4i\hbar \left[\sum_{n=1} \frac{(-1)^{n-1}}{2^n n} \left(\Lambda \overleftarrow{U}^2 \right)^{n-1} \Lambda \right]. \quad (35)$$

As a result, the corresponding Ward identity, with the FD parameters $\mu_a(F') = \frac{i}{2\hbar} g(y) F' \overleftarrow{U}_a$, $\Lambda(\Gamma|F') = \frac{i}{2\hbar} g(y) F'$, for $y \equiv (i/4\hbar) F' \overleftarrow{U}^2$, and the gauge-dependence problem acquire the form [5]

$$\left\langle \left\{ 1 + \frac{i}{\hbar} J_A \phi^A \left[\overleftarrow{U}^a \mu_a(\Lambda) + \frac{1}{4} \overleftarrow{U}^2 \mu^2(\Lambda) \right] - \frac{1}{4} \left(\frac{i}{\hbar} \right)^2 J_A \phi^A \overleftarrow{U}^a J_B (\phi^B) \overleftarrow{U}_a \mu^2(\Lambda) \right\} \right. \\ \left. \times \left(1 - \frac{1}{2} \Lambda \overleftarrow{U}^2 \right)^{-2} \right\rangle_{F, J} = 1, \quad (36)$$

$$Z_{F+F'}(J) = Z_F(J) \left\{ 1 + \left\langle \frac{i}{\hbar} J_A \phi^A \left[\overleftarrow{U}^a \mu_a(\Gamma| - F') + \frac{1}{4} \overleftarrow{U}^2 \mu^2(\Gamma| - F') \right] \right. \right. \\ \left. \left. - (-1)^{\varepsilon_B} \left(\frac{i}{2\hbar} \right)^2 J_B J_A \left(\phi^A \overleftarrow{U}^a \right) \left(\phi^B \overleftarrow{U}_a \right) \mu^2(\Gamma| - F') \right\rangle_{F, J} \right\}, \quad (37)$$

with source-dependent average expectation value with respect to $Z_F(J)$ corresponding to a gauge-fixing $F(\phi)$.

By choosing the $N = 1$ or $N = 2$ SB BRST symmetry term $M(\phi)$ as the horizon functional $H(A)$ in Landau (or Coulomb) gauge, and assuming the gauge independence of the path integrals $Z_{H, \psi}$, $Z_{H, F}$ under a finite change of the gauge condition, $\psi \rightarrow \psi + \psi'$ or $F \rightarrow F + F'$, one can determine the functional $H(A)$ in a new reference frame, $\psi + \psi'$ or $F + F'$, of the respective $N = 1, 2$ BRST symmetry setting, with account taken of (33), (35):

$$H_{\psi'}(\phi) = H(A) \{1 + \overleftarrow{s} \mu(\psi')\} \text{ or } H_{F'}(\phi) = H(A) \left\{ 1 + \overleftarrow{s}^a \mu_a(F') + \frac{1}{4} \overleftarrow{s}^2 \mu^2(F') \right\}. \quad (38)$$

Notice in conclusion that the above $N = 1, 2$ FD BRST transformations make it possible to study their influence on the Yang–Mills, Gribov–Zwanziger, Freedman–Townsend models, and the Standard Model, as well as on the concept of average effective action [1, 2, 3, 5, 6]. The case of functionally-dependent parameters $\mu_a(\Gamma) = \Lambda(\Gamma) \overleftarrow{s}^a + \psi_a(\Gamma)$ with a vanishing BRST-antiBRST “divergence”, $\psi_a \overleftarrow{s}^a = 0$, was examined in Sec. 4.1. of Ref. [6] and implies a modified form of the compensation equation, due to a nontrivial contribution of ψ_a to the corresponding Jacobian.

Acknowledgments The authors are grateful to the organizers of the International Workshop SQS’15 for kind hospitality, as well as to M.O. Katanaev and K.V. Stepanyantz for their interest and discussions. The study was supported by the RFBR grant No. 16-42-700702, and was also partially supported by the Tomsk State University Competitiveness Improvement Program.

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